Analysis of Bonded Joints in Vehicular Structures

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A linear elastic finite-element analysis is used to evaluate the behavior of an adhesive joint bonding an advanced composite to a metallic substrate. The adhesive layer is treated as a separate elastic medium of finite thickness in order to remove the stress singularity that exists when dissimilar materials are joined. Results are presented for a single lap joint and a smoothly tapered joint for two thicknesses of the adhesive layer. For the lap joint, peak shearing stress in the adhesive is located adjacent to the material with the lower modulus. The smoothly tapered joint is shown to have lower stresses and to be more efficient. Recommendations are given for alleviating stress concentrations and improving the design of bonded joints.

Introduction

RECENT developments in the application of advanced composites to vehicle design involve material systems utilizing composites bonded to metallic structural members. These hybrid-composites combine the high tensile capacity of a composite with the shear strength of a metal to obtain a structurally efficient and cost effective component. Applications have included horizontal stabilizers of aircraft, tail sections of helicopters, and high speed ground transportation vehicles.

To extend the use of composites as reinforcing elements in vehicular structures, the transfer of stress between the composite and the metallic substrate must be understood. The existence of a stress singularity at the intersection of the free edge and the interface when dissimilar materials are bonded has been demonstrated by Bogy and Wang.[1] In a study by Erdogan and Ratwani[2] of stepped joints, the adhesive layer was considered as a separate elastic medium and the stress singularity at the end of the joint was removed. By assuming a state of generalized plane stress, they obtained a closed form solution for the stress distribution in joints between two plates.

In this paper, a two-dimensional analysis based on the finite-element method is used to determine the stress state throughout the transition region surrounding a bonded joint. A special element developed by Goodman, Taylor, and Brekke[3] is used to represent the adhesive layer in the joint as a thin elastic medium. A general isoparametric quadrilateral[4] which is compatible with the adhesive element is used in the idealization of the two plates being joined. In Fig. 1 are shown a single lap joint and a smoothly tapered joint under uniaxial extension. The analysis of these joints for different adhesive thicknesses and plant materials is discussed herein.

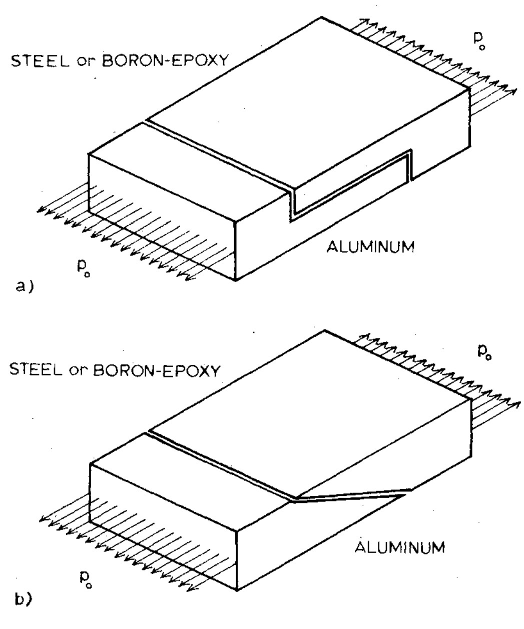

Fig. 1 Bonded joint configurations: a) single lap joint; b) smoothly tapered joint.

Analysis

The finite-element analysis used is linear elastic and considers only static inplane loads. A brief description of the elements and the idealization utilized in modelling the bonded joint are given in this section.

General Material Element

A two-dimensional plane stress-plane strain element is used to represent the material in the two plates being joined. This general quadrilateral element is shown in Fig. 2 and is a member of the isoparametric family introduced by Ergatoudis, Irons, and Zienkiewicz.[4] The element has two inplane displacements at each of its four nodes to give a total of eight degrees-of-freedom per element. The inplane displacements are expressed by a bilinear form in terms of the generalized coordinates α as

$$u(\xi, \eta) = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta + \alpha_4 \xi \eta \tag{1}$$

$$v(\xi, \eta) = \alpha_5 + \alpha_6 \xi + \alpha_7 \eta + \alpha_8 \xi \eta \tag{2}$$

where ξ and η identify the skew coordinate system given in Fig. 2. This displacement pattern results in straight line deformation along element boundaries and continuity between elements exists.

Material properties are specified as either isotropic or orthotropic depending on whether a metallic or composite plate is being idealized. Both plane stress ($\sigma_z = 0$) and plane strain ($\varepsilon_z = 0$) can be analyzed if a comparison between these two states of stress is desired.

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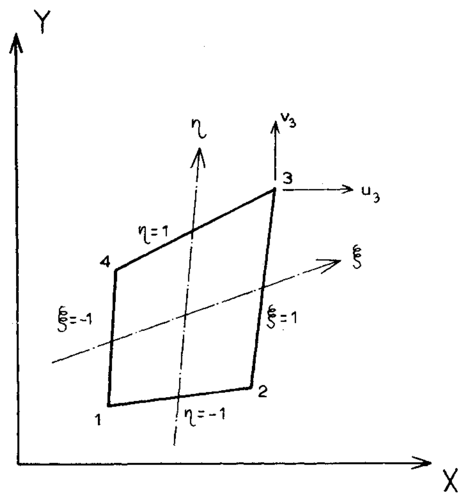

Fig. 2 General material element.

Adhesive Element

A special element is used to model the adhesive layer between the two plates as a separate elastic medium. It is not possible to simply shrink the quadrilateral element until it is thin in one direction, because the stiffness coefficients associated with the thin direction then become dominant and numerical ill-conditioning results.

The four node rectangular element developed by Goodman, Taylor, and Brekke[3] is shown in Fig. 3 for an arbitrary orientation with respect to the reference or global axes. The element has length, L, and a finite thickness, h, defined in its local coordinate system. When a joint is idealized, the nodal point pairs (1, 4) and (2, 3) are given identical coordinates regardless of the thickness of the adhesive layer. Thus, the joint element is acting like a combined normal and shear spring between the two plates, not unlike that used in the analysis of beams on an elastic foundation.

In the local coordinate system, a linear displacement pattern is given in terms of the nodal displacements along the top of the element as

$$u^{\text{top}}(\xi) = 1/2[(1+\xi)u_3] + 1/2[(1-\xi)u_4] \tag{3}$$

$$v^{\text{top}}(\xi) = 1/2[(1+\xi)v_3] + 1/2[(1-\xi)v_4] \tag{4}$$

where $\xi = 2x/L$. A similar relationship is written for the displacements along the bottom so that the relative displacement becomes

$$w_s = 1/2[-(1-\xi)u_1 - (1+\xi)u_2 + (1+\xi)u_3 + (1-\xi)u_4] \tag{5}$$

$$w_n = 1/2[-(1-\xi)v_1 - (1+\xi)v_2 + (1+\xi)v_3 + (1-\xi)v_4] \tag{6}$$

where w_s = relative tangential displacement and w_n = relative normal displacement. Equations (5) and (6) can be rewritten in matrix form as

$$\{w\} = [B]\{u\} \tag{7}$$

where $\{w\}^T = [w_s, w_n]$ and $\{u\}^T = [u_1, u_2, \ldots, v_1, \ldots, v_4]$. Here it becomes apparent that $[B]$ represents the strain-displacement matrix.

The material properties for the element relate the force per unit length to the relative displacement by the relations

$$P_s = k_s w_s \tag{8}$$

$$P_n = k_n w_n \tag{9}$$

where P = force per unit length, k = adhesive stiffness per unit length, and the subscripts s and n refer to the tangential and normal directions, respectively. The units of k_s and k_n are force per unit of area and are related to the moduli of elasticity of the adhesive by

$$k_s = (G/h)L \tag{10}$$

$$k_n = (E/h)L \tag{11}$$

where G = shear modulus of the adhesive and E = Young's modulus of the adhesive.

In order to obtain the element stiffness matrix $[k]$ the following integration has to be carried out:

$$[k] = \int_0^L [B]^T[D][B]\,dx \tag{12}$$

$[D]$ represents the elasticity matrix and is of the following form:

$$[D] = \begin{bmatrix} k_s & 0 \\ 0 & k_n \end{bmatrix} \tag{13}$$

This leads to the explicit form of the element stiffness matrix given in Ref. 3

$$[k] = \begin{bmatrix} 2k_s & 0 & k_s & 0 & -k_s & 0 & -2k_s & 0 \\ & 2k_n & 0 & k_n & 0 & -k_n & 0 & -2k_n \\ & & 2k_s & 0 & -2k_s & 0 & -k_s & 0 \\ & & & 2k_n & 0 & -2k_n & 0 & -k_n \\ & & & & 2k_s & 0 & k_s & 0 \\ & \text{Symmetric} & & & & 2k_n & 0 & k_n \\ & & & & & & 2k_s & 0 \\ & & & & & & & 2k_n \end{bmatrix} \tag{14}$$

The rotation of the coordinate axes from the local to the global coordinate system can be performed in the following way

$$[K] = \begin{bmatrix} R^T & & & \\ & R^T & & \\ & & R^T & \\ & & & R^T \end{bmatrix} [k] \begin{bmatrix} R & & & \\ & R & & \\ & & R & \\ & & & R \end{bmatrix} \tag{15}$$

where

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

and the angle θ as indicated in Fig. 3.

Idealization

The general quadrilateral and adhesive elements both have two inplane displacements per node and can be combined without difficulty. Moreover, they both have linear displacement patterns along their boundaries so that compatibility between elements is preserved, and available solution bounds are valid.

Idealizations of the two joint configurations studied are shown in Fig. 4. A relatively large number of elements are placed near the adhesive joint where the stresses are rapidly changing. A coarser grid is used away from the joint with the transition conveniently handled by allowing two nodes of the general

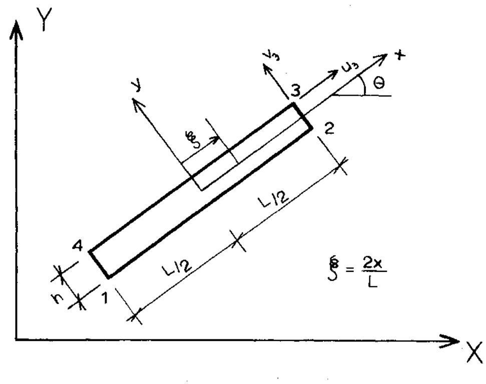

Fig. 3 Adhesive element.

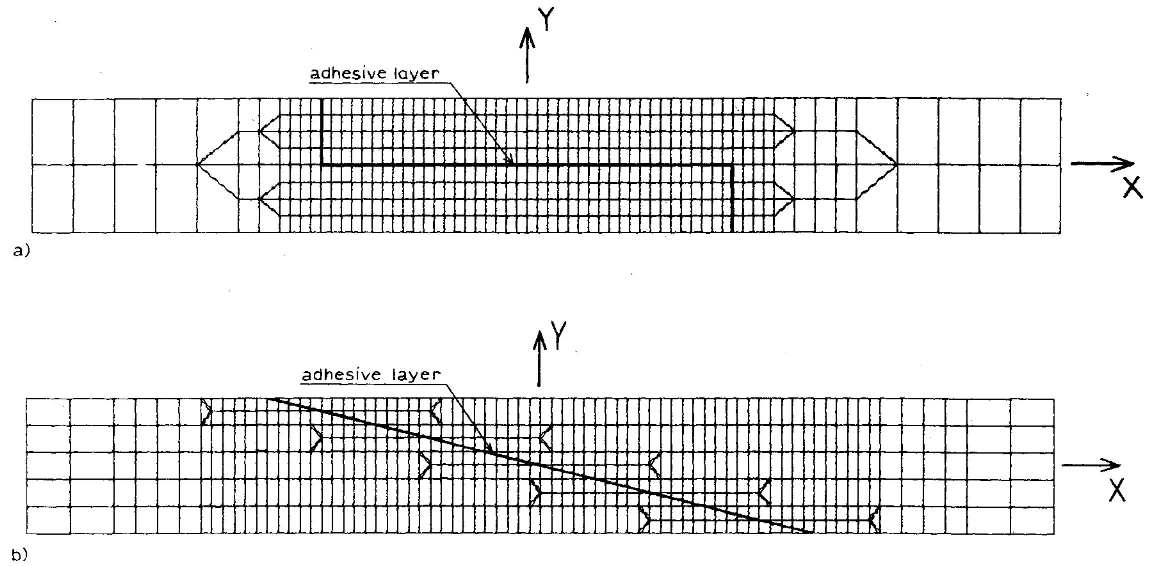

Fig. 4 Finite-element idealizations: a) single lap joint; b) smoothly tapered joint.

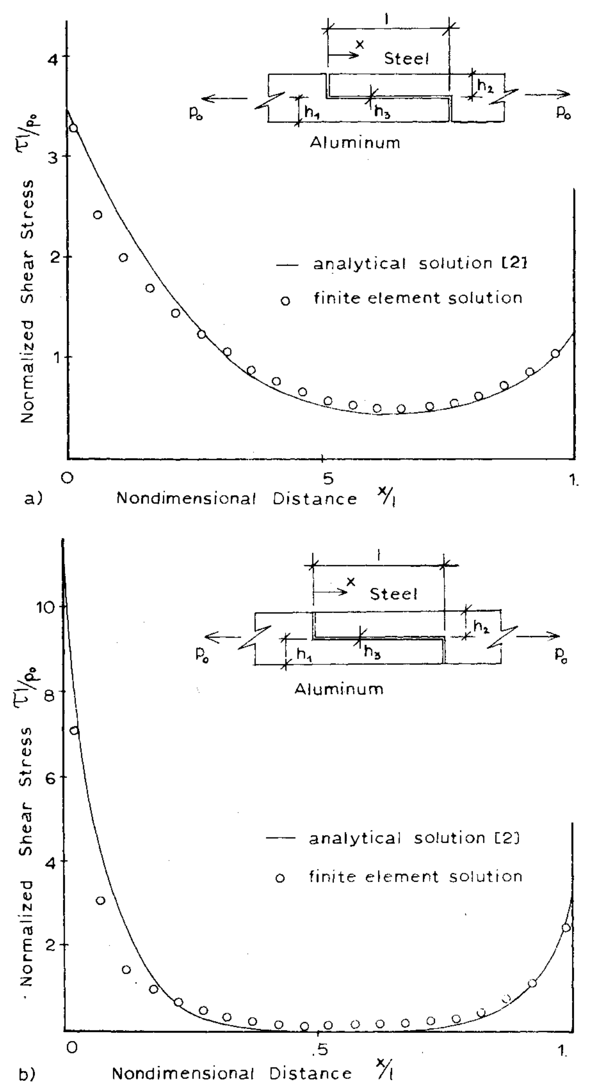

Fig. 5 Shear stress in a single lap joint, aluminum/steel, $h_1 = h_2 = 0.1l$: a) adhesive thickness, $h_3 = 0.01l$; b) adhesive thickness, $h_3 = 0.001l$.

quadrilateral to coincide to form the triangular elements shown.

Uniform extension is applied to the model by displacing the ends a finite amount in the X-direction. If the surfaces perpendicular to the Y-axis are left free, the difference in stiffness of the plates being joined produces bending in the X-Y plane. In order to compare with the generalized plane stress solution of Ref. 2, it was necessary to restrain the bottom surface of the plates on roller supports to remove the bending distortions.

Single Lap Joint

Results for the single lap joint configuration of Figs. 1a and 4a are presented in this section for two different material combinations: aluminum/steel and aluminum/boron-epoxy. Both shear stresses in the adhesive and normal stresses in the base materials are given for two different thicknesses of the adhesive layer.

Aluminum/Steel

To evaluate the ability of the analysis developed in the previous section to determine stress distributions, the finite-element results are compared with the closed form solutions of Ref. 2 in Fig. 5. The material properties and dimensions used are

Aluminum: $E_1 = 10 \times 10^6$ psi, $\nu_1 = 0.3$, $h_2 = 0.1l$

Steel: $E_2 = 30 \times 10^6$ psi, $\nu_2 = 0.3$, $h_2 = 0.1l$

Adhesive (epoxy): $E_3 = 4.45 \times 10^5$ psi, $G_3 = 1.65 \times 10^5$ psi, $h_3 = 0.01l$ and $0.001l$

In order to compare with the analysis of Ref. 2, it is necessary to remove the adhesive element from the vertical portions of the joint. The model then represents a bonded joint that has failed in tension at the ends of the joint and is transferring the load through shear of the adhesive along the horizontal contact surfaces. The agreement between the analytical solution and the finite-element analysis is excellent for both adhesive thicknesses.

The applied force, p_o, is in units of pounds and the shear stress resultant, τ, is in units of pounds per inch. To make the curves plotted in Fig. 5 nondimensional, the shear stress resultant has been multiplied by the length of the joint and divided by the applied force. The distance parameter has also been normalized by dividing the x location by the length of the joint. Thus, integrating the shear stress from one end of the joint to the other gives the area under the curve as unity.

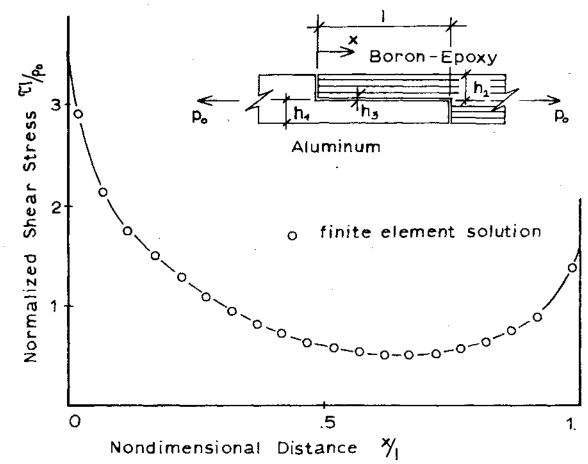

Fig. 6 Shear stress in a single lap joint, aluminum/boron-epoxy, $h_1 = h_2 = 0.1l$, $h_3 = 0.01l$.

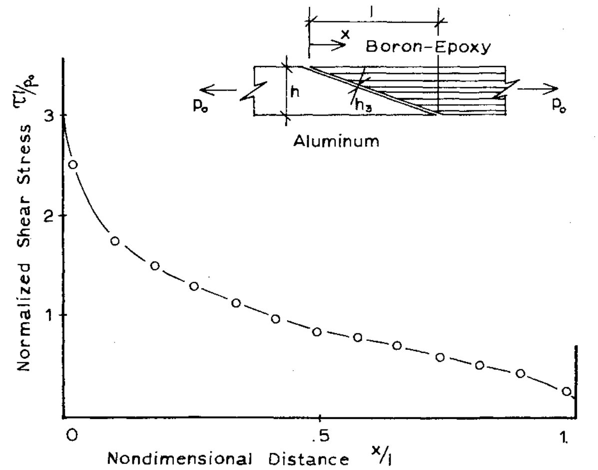

Fig. 8 Shear stress in a smoothly tapered joint, aluminum/boron-epoxy, $h = 0.2l$, $h_3 = 0.001l$.

Two peak shear stresses are observed at the ends of the lap joints in Fig. 5, with the larger value occurring at $x = 0$. These peak stresses increase when the thickness of the adhesive layer is decreased. Also, the rate of decay of the peak stresses increases with decreasing adhesive thickness. This is necessary because the area under the curves for the two different thicknesses must remain constant for the normalized stresses and dimensions given.

Aluminum/Boron-Epoxy

A second example combines an isotropic metal and an orthotropic composite through an epoxy adhesive single lap joint. The material properties utilized are

Aluminum: $E_1 = 10 \times 10^6$ psi, $\nu_1 = 0.3$, $h_1 = 0.1l$

Boron-Epoxy: $E_{2x} = 32.4 \times 10^6$ psi, $\nu_{2x} = 0.23$

$E_{2y} = 3.5 \times 10^6$ psi, $\nu_{2y} = 0.3$

$G_2 = 1.23 \times 10^6$ psi, $h_2 = 0.1l$

Adhesive: $E_3 = 4.45 \times 10^5$ psi, $G_3 = 1.65 \times 10^5$ psi, $h_3 = 0.01l$

The shear stress for this case is plotted in Fig. 6 and shows a variation similar to the aluminum/steel results (Fig. 5a) for the same adhesive thickness. This is not unexpected because the boron-epoxy has about the same tension modulus as steel in the direction of its fibers and the orthotropic properties do not have a strong influence. If the plate is restrained in the thickness direction and the vertical ends of the joint are filled with adhesive a larger difference would be observed when the orthotropic material is used.

The variations of the normal stresses in the two materials on either side of the joint are shown in Fig. 7 for the same problem as Fig. 6. Single peak stresses of about equal magnitude exist for each material. The maximum value for the aluminum is at $x = 0$, while the maximum for the boron-epoxy is at $x = l$. These maximums give a stress concentration factor of about 3 when compared to a uniform stress away from the joint.

Smoothly Tapered Joint

The analysis is also applied to the smoothly tapered joint configuration of Figs. 1b and 4b. Results are presented for the aluminum/boron-epoxy material combination and an adhesive thickness of $0.001l$. The total thickness of the plate is $0.2l$, the same as in the previous examples.

Figure 8 shows the shear stress variation in the adhesive layer. It has one peak stress at $x = 0$, and decreases monotonically to a minimum value at $x = l$. With the oblique joint,

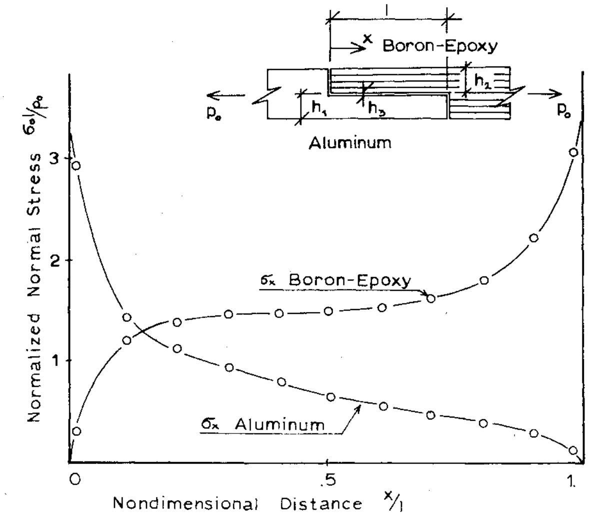

Fig. 7 Normal stress in a single lap joint, aluminum/boron-epoxy, $h_1 = h_2 = 0.1l$, $h_3 = 0.01l$.

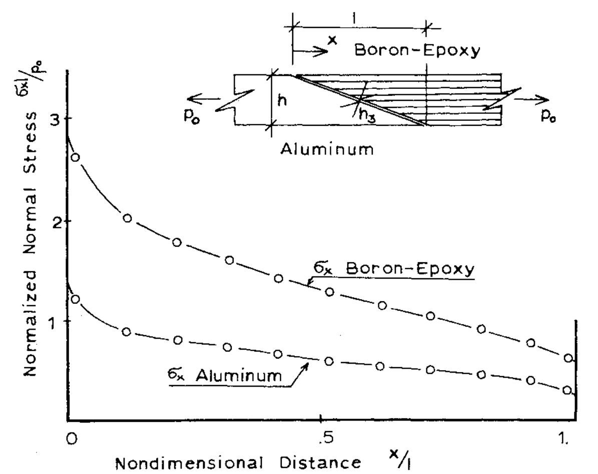

Fig. 9 Normal stress in a smoothly tapered joint, aluminum/boron-epoxy, $h = 0.2l$, $h_3 = 0.001l$.

the load transfer is accomplished by a combination of tensile and shear forces. This results in a maximum normalized shear stress of less than 3 compared to a value greater than 10 for the lap joint (Fig. 5b) with the same adhesive thickness of $0.001l$. Thus, the smoothly tapered joint is more efficient in reducing the peak shear-stresses than the single lap joint.

The normal stresses in the materials on either side of the smoothly tapered joint are plotted in Fig. 9 for the same example as Fig. 8. The peak normal stress occurs in both materials at $x = 0$ and monotonically decreases to a minimum at $x = l$. This is in contrast to the double peaks and the increasing-decreasing behavior in Fig. 7 for the single lap joint. The highest stress concentration occurs in the stiffer material (boron-epoxy) and has a magnitude of 3. This is comparable to that found in a single lap joint with the thicker adhesive layer (Fig. 7).

Discussion and Conclusions

A valid finite-element analysis of bonded joints has been presented. A special feature of this analysis is the modeling of the adhesive layer with a joint element which behaves as a combined shear and tension spring whose stiffness properties vary with thickness. By use of the joint element, the adhesive layer is treated as a separate elastic medium and stresses throughout the transition region in the neighborhood of the joint remain finite.

When the load transfer mechanism is basically shear, as in a lap joint, high shear stresses occur in the adhesive near the ends of the joint. When the thickness of the adhesive layer is decreased, these shear stresses become even larger. Thus, debonding of a joint will probably occur at its ends when the adhesive layer is thin.

When the load transfer mechanism is a combination of shear and tension as in a smoothly tapered joint, the peak shear stresses in the adhesive are reduced and occur at only one end of the joint. The maximum normal stresses in the base material are also less than those in a lap joint. Therefore, a smoothly tapered joint is recommended for bonding composite elements to metallic substrates.

With the analysis procedure established, additional studies can be conducted on improving the geometry of the joint to reduce stress concentrations. Variations in the angle of taper and the removal of sharp corners should be investigated. Adhesives with different shear moduli should be considered and their influence on joint deformation evaluated.

Future work should be addressed to failure criteria and strength predictions of bonded joints. Interlaminar shear stress in the composite material may be as critical as the shear stress in the adhesive. The relative shear strengths of these two materials will determine where the failure in the joint is likely to occur. Once this is known then measures can be taken to improve the behavior of the joints and the over-all response of the structural component.

References

[1] Bogy, D. B. and Wang, K. C., "Stress Singularities at Interface Corners in Bonded Dissimilar Isotropic Elastic Materials," *International Journal of Solids and Structures*, Vol. 7, 1971, pp. 993–1005.

[2] Erdogan, F. and Ratwani, M., "Stress Distribution in Bonded Joints," *Journal of Composite Materials*, Vol. 5, July 1971, pp. 378–393.

[3] Goodman, R. E., Taylor, R. L., and Brekke, T. L., "A Model for the Mechanics of Jointed Rock," *Journal of the Soil Mechanics and Foundations Division, ASCE*, Vol. 93, SM3, 1968, pp. 637–659.

[4] Ergatoudis, J. G., Irons, B. M., and Zienkiewicz, O. C., "Curved Isoparametric, Quadrilateral Elements for Finite Element Analysis," *International Journal of Solids and Structures*, Vol. 4, 1968, pp. 31–42.